Instituto Superior de Ciencias y Tecnología Nucleares

Dissertation Master of Science Thesis

Gluon, Quark and Hadron Masses from a Modified Perturbative QCD

Author: Marcos Rigol Madrazo

Advisor: Dr. Alejandro Cabo Montes de Oca

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Abstract

The development of a Modified Perturbation Theory for QCD, introduced in previous works, is continued. The gluon propagator is modified as consequence of a soft gluon pairs condensate in the vacuum. The modified Feynman rules for $\alpha = 1$ are shown, and some physical magnitudes calculated with them. The mean value of G^2, gluon masses and the effective potential are calculated up to the g^2 order, improving previous calculations. In connection with the gluon self-energy it follows that the gluonic mass shell becomes tachyonic in the considered approximation. The constituent quarks masses, produced by the influence of the condensate, are also calculated. Results of the order of 1/3 of the nucleon mass, are obtained for the constituent masses of the up and down quarks. In addition, the predicted flavour dependence of the calculated quarks masses turns out to be the appropriate to reproduce the spectrum of the ground states within the various groups of hadronic resonances, through the simple addition of their constituent quark masses. Finally, the generalization of the theory for arbitrary values of the α gauge parameter is explored.

Contents

Chapter 1

Introduction

The development of Quantum Chromodynamics (QCD) has been one of the greatest achievements of the High Energy Theoretical Physics in the latest thirty years. As it is well known the smallest of the coupling constant at high momenta (asymptotic freedom), made possible the development of the theory through the usual perturbative language. That situation strongly simplifies the study of such phenomena and the calculated results are in very good agreement with the experimental data. However, this so called perturbative QCD (PQCD) is far from being able to furnish even a rough description for the relevant physics at low energies. The solution of this situation is currently one of the main challenges of the Theoretical Physics.

A relevant phenomenon related with QCD is the color confinement. For the time being, there are strong reasons to believe that the relation between the basic quantities in QCD, the colored gluon and quarks fields, and the real world characterized by a whole variety of colorless interacting mesons and baryons, can be understood by solving the confinement problem. The basic picture is that a theory in which $SU(3)$ is an exact symmetry, the fundamental fields (quarks and gluons) cannot be associated to physical states and then the true physical states consist only of colorless mesons and baryons.

The above cited limitations of the PQCD means in particular that the usual Fock space vacuum for the non-interacting theory, is unable to predicts even approximately the real ground state of the QCD [1]-[4]. This is at contrast with the case in QED where the standard perturbation theory, based in the empty of the Fock space, gives a more than good concordance with the experimental results. Then, the nature of the vacuum structure is one of the main problems to be clarified and naturally its solution is closely related with the color confinement effect. A review of the various models considered to investigate the confinement and the vacuum structure in QCD can be found in Refs. [5, 6].

Another important problem for QCD is the one related with quark masses. There are no free quarks, so it is necessary to take care with the meaning of quark masses. They should be understood as mass parameters appearing in the QCD Lagrangian [7]-[9] (called current quark masses) and their origins have generated polemic for long time [10]. The theoretical ratios between the current quark masses and their absolute values can be found with the standard methods of current algebra. Their experimental values are determined indirectly and the most recently reports for them [11] will be used in the present work. On the other hand the masses considered in phenomenological hadron models are called constituent quark masses, which values are higher than the ones of current masses, and it is supposed that such values come from non-perturbative effects, like gluonic condensate effects for example. Other properties of quarks, like magnetic moments, couplings, and electro-weak form factors are not influenced by this non-perturbative effects [12].

In the former works [13]-[15] an attempt to construct a modified perturbation expansion for QCD, able to predict some low energy properties of this theory, have been considered. The proposed modified expansion [13] conserves the color $SU(3)$ and Lorentz symmetries, and was implemented in order to solve the symmetry limitations of the earlier chromomagnetic field models [1]-[4] which inspired the search. The similar lack of manifest Lorentz invariance had also a modification of the Feynman rules, attempting to include gluon condensation, advanced later [16] in which a delta function term, for $k < \Lambda_{qcd}$, was considered in addition to the perturbative piece. The expansion proposed in a previous work [13], considers a change of the gluon propagator in a term associated to a condensation of zero momentum gluons. This change of the usual rules had the interesting property of producing a non vanishing value for the gluon condensation parameter $\langle g^2G^2\rangle$ [17] already in the first approximations. In addition, in Ref. [13] a non vanishing value for the effective mass of the gluons was obtained at the one loop level. Finally, a perturbative evaluation of the effective potential as a function of the condensate parameter indicated that the condensation is spontaneously generated from a zero condensate state.

At this point it is important to remark that earlier works [18, 19] introduced a pure delta function at zero momentum as gluon propagator, searching for a model of meson resonances. In these cases the constants introduced, multiplying the delta function, were different to the one fixed in our previous work [14, 15], which lead to also different results. In particular, the singularity structure of the quark propagator have no pole on the real p^2 axis [19] at difference with our results.

Justifying the applicability of the Feynman expansion introduced in Ref. [13] was advanced later [14, 15]. This was in need because after modifying the propagator, it was unknown whether or not the initial state generating the expansion was a physical state of the theory. In Ref. [14, 15], considering the operational formulation of QCD developed by Kugo and Ojima [20, 21], was possible to find a zero momentum gluon condensate state able to generate through the Wick expansion the sort of propagators considered in Ref. [13]. It was checked that this condensate satisfies the physical state conditions imposed by the BRST formalism. The discussion allowed also to a more precise characterization of the class of changes admitted in the diagramatic expansion by the physical state conditions on the initial state. Specifically it was found that the C parameter, describing the gluon condensation, must be real and positive. It should be remarked that an analogous proposal for a zero mode gluon condensate state was independently advanced in Ref. [22], in an attempt to give foundation to the propagator proposed by Munczek and Nemirovski.

In the present work the development of the Modified Perturbation Theory [13]-[15] is continued, and the objectives proposed are: a) To state clearly the details of the considered perturbative expansion for $\alpha = 1$. b) To realize an improved evaluation of quantities calculated in Ref. [13], that is, the mean value of G^2, the gluon mass, and the effective potential, by also imposing the restriction that appeared on the parameter C describing the condensation [14, 15]. c) To apply the results of the calculation of $\langle G^2 \rangle$ to evaluate the effects of the condensate on the quark masses, after fixing the parameter $g^2 C$ to reproduce the accepted value for $\langle g^2 G^2 \rangle$. d) Finally, to explore the generalization of the theory for arbitrary values of the gauge parameter α.

The evaluations presented here are done in a special approximation. It can be defined as the one allowed to be considered with precision by the fact that in the present state of the analysis the renormalization of the fields and coupling constants has not being introduced yet. Therefore, the calculations of the $\langle G^2 \rangle$, the quark and gluon masses will be searched not only up to the order g^2 but also with the additional restriction of disregarding all the terms not involving the condensate contribution. To take into account those terms will need the introduction of the renormalization procedure up to two loops, because the one loop divergence are unchanged by the use of the new propagator. This is out of the scope of the present exploration. It is worth noting that the just defined approximation scheme resembles a sort of quasi-classical limit in which the condensate would play the role of a macroscopic quantum field. The correctness of such a picture is suggested by the initial motivation of the analysis, as directed to construct a covariant version of the chromomagnetic models [13].

The exposition will be organized as follows: Chapter 2 is divided in two sections. In the first one a review of the previous work [14, 15] is done. In the second section the Feynman Rules for the considered perturbative expansion ($\alpha = 1$) are stated. Chapter 3 is divided in four sections. In the first one the mean value of G^2 is calculated and the free parameter $g^2 C$ fixed from the accepted value of $\langle g^2 G^2 \rangle$. In the second and third sections the gluon and quark masses respectively are determined. In the four section the effective potential is evaluate as a function of $\langle G^2 \rangle$. Chapter 4 is devoted to explore the generalization of the theory for arbitrary values of the gauge parameter α. Finally, two appendixes are introduced. In the first one it is considered the exposition of the diagrams calculated for determining the mean value of G^2. The second one was introduced for an analysis of the most elaborated parts in the calculation of longitudinal and scalar modes contribution to the gluon propagator modification, considered in Chapter 4.

Chapter 2

Modified Feynman Rules

The Modified Feynman rules, advanced in previous works, are stated. The gluon propagator is modified as consequence of a soft gluon pairs condensate in the QCD vacuum, this fact is represented by a delta function in momentum representation. In the first section the previous work is shortly reviewed and in the second section the Feynman rules exposed.

2.1 Review of "About an Alternative Vacuum State for Perturbative QCD"

In the previous work [14, 15] the construction of a modified vacuum state for perturbative QCD was considered. By using the operational formulation for the Quantum Gauge Field Theory developed by Kugo and Ojima, a particular state vector for QCD in the non-interacting limit, that obeys the BRST physical state condition, was proposed. The general motivation for searching this wave function was to give a foundation to the perturbative expansion proposed in Ref. [13]. The proposed state had the form

$$| \widetilde{\Phi}\rangle = \frac{1}{\sqrt{N}} \exp \sum_a \left(\frac{1}{2} A^{a+}_{0,1} A^{a+}_{0,1} + \frac{1}{2} A^{a+}_{0,2} A^{a+}_{0,2} + B^{a+}_0 A^{L,a+}_0 + i \overline{c}^{a+}_0 c^{a+}_0 \right) | 0\rangle. \tag{2.1}$$

The state (2.1) is a coherent superposition of zero momentum gluon pairs, expressed in terms of the transverse and longitudinal gluon $A^{a+}_{0,1}$, $A^{a+}_{0,2}$, $A^{L,a+}_0$, ghosts c^{a+}_0, $\overline{c}^{a+}_0$, and Nakanishi-Lautrup B^{a+}_0 creation operators of the free theory [20, 21]. This state is colorless because the contracted color index a. Eq. (2.1) should be understood in a sense of limit. The operators are supposed to be evaluated at the smallest non-vanishing null momentum $\vec{p}$ resulting from quantizing the QCD free fields with periodic boundary conditions in a large box of volume $V \to \infty$. The coefficients of the products of creation operators are slightly different from their shown numerical values before the infinity volume limit is taken. They reach to the values in expression (2.1) when $V \to \infty$ $(p \to 0)$ in the way determined previously [14, 15].

For the construction of the state (2.1), the BRST physical state conditions

$$\begin{aligned} Q_B \mid \Phi\rangle &= 0, \\ Q_C \mid \Phi\rangle &= 0, \end{aligned} \tag{2.2}$$

were required, in which Q_B and Q_C are the charges associated to the BRST symmetry and ghost number conservation. The operational quantization was considered in a scheme of the Gupta-Bleuler type, in which all the gluons are treated on the same footing [20, 21]. This corresponds with the selection of the gauge parameter $\alpha = 1$ in the standard functional approach. In justifying proceeding in such a way, the central assumption was that under the adiabatic connection of the color interaction, the evolution won't bring the state out of the physical subspace at any stage of the connection. Thus, the final state will be also a physical one for the interacting theory and the associated Feynman expansion should have a physical meaning. It is clear that this approach left out the question of the construction of a gauge parameter independent formulation of the theory. However, as a minimal logical ground for the physical relevance of the predictions was given, the generalization of the analysis for and arbitrary α was postponed. In Chapter 4 of the present work this generalization is explored.

The gluon propagator corresponding to the modified vacuum state (2.1) was calculated [14, 15] through the expression for the generating functional of the free particle Green functions $Z_0\left(J\right)$ [23],

$$\begin{aligned} &\langle\widetilde{\Phi} \mid \exp\left\{i\int d^4x \sum_{a=1,..,8} J^{\mu,a}\left(x\right) A_\mu^{a-}\left(x\right)\right\} \exp\left\{i\int d^4x \sum_{a=1,..,8} J^{\mu,a}\left(x\right) A_\mu^{a+}\left(x\right)\right\} \mid \widetilde{\Phi}\rangle \\ &\times \exp\left\{\frac{i}{2}\sum_{a,b=1,..8}\int d^4x d^4y J^{\mu,a}\left(x\right) D_{\mu\nu}^{ab}(x-y) J^{\nu,b}\left(y\right)\right\} \\ &= \exp\left\{\frac{i}{2}\sum_{a,b=1,..8}\int d^4x d^4y J^{\mu,a}\left(x\right) \widetilde{D}_{\mu\nu}^{ab}(x-y) J^{\nu,b}\left(y\right)\right\}, \end{aligned} \tag{2.3}$$

where the usual gluon propagator is denoted by $D_{\mu\nu}^{ab}(x-y)$ and $\widetilde{D}_{\mu\nu}^{ab}(x-y)$ is the modified one.

The result obtained for the modified propagator in the momentum representation was,

$$\widetilde{D}_{\mu\nu}^{ab}(k) = \delta^{ab} g_{\mu\nu} \left[\frac{1}{k^2} - iC\delta(k) \right], \tag{2.4}$$

In Eq. (2.4), C is the parameter associated with the gluon condensate, and was determined to be real and non-negative [14, 15]

The modification to the standard free ghost propagator, introduced by the proposed initial state, was also calculated. Moreover, after considering the free parameter in the proposed trial state as real (2.1), the ghost propagator was not modified.

2.2 The Generating Functional and the Modified Feynman Rules, for $\alpha = 1$

In the present section the modified Feynman rules for the theory are exposed. The generating functional of the Green functions, can be written as

$$Z\left[J, \bar{\eta}, \eta, \bar{\xi}, \xi\right] = \frac{\int D\left(A, \bar{c}, c, \bar{\Psi}, \Psi\right) \exp\left\{i \int d^4x \left(\mathcal{L} + J^{\mu,a} A_\mu^a + \overline{c}^a \eta^a + \bar{\eta}^a c^a + \bar{\Psi}^i \xi^i + \bar{\xi}^i \Psi^i\right)\right\}}{\int D\left(A, \bar{c}, c, \bar{\Psi}, \Psi\right) \exp\left\{i \int d^4x \mathcal{L}\right\}}, \tag{2.5}$$

where sources have been introduced for all the fields in the usual manner, and the effective Lagrangian is given by [20, 21]

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_G + \mathcal{L}_{Gh} + \mathcal{L}_Q & (2.6)\\
\mathcal{L}_G &= -\frac{1}{4} F_{\mu\nu}^a(x) F^{\mu\nu,a}(x) - \frac{1}{2\alpha}\left(\partial^\mu A_\mu^a(x)\right)^2, & (2.7)\\
\mathcal{L}_{Gh} &= -i\partial^\mu \overline{c}^a(x) D_\mu^{ab}(x) c^b(x), & (2.8)\\
\mathcal{L}_Q &= \bar{\Psi}^i(x)\left(i\gamma^\mu D_\mu^{ij} - m_Q \delta^{ij}\right)\Psi^j(x). & (2.9)
\end{aligned}$$

The sum over the six quark flavors will be omitted everywhere in order to simplify the exposition, no confusion should arise for it. The gluon field intensity has the usual form

$$F_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + g f^{abc} A_\mu^b(x) A_\nu^c(x),$$

D_μ^{ab}, D_μ^{ij} are the covariant derivatives in the adjoint and fundamental representations respectively, for the $SU(3)$ group

$$\begin{aligned} D_\mu^{ab}(x) &= \partial_\mu \delta^{ab} - g f^{abc} A_\mu^c(x), \\ D_\mu^{ij}(x) &= \partial_\mu \delta^{ij} - i g T^{ij,a} A_\mu^a. \end{aligned}$$

It should be recalled that as argued in [14, 15] the physical predictions for the value $\alpha = 1$ of the gauge parameter should have physical meaning whenever the adiabatic connection of the interaction does not lead the evolving state out the physical space. Therefore here this value of the parameter will be fixed for this Chapter and the next one. The generalization is advanced in Chapter 4.

Then in (2.5), the standard procedure of extracting out of the functional integral the exponential of terms higher than second order in the fields (vertexes) can be used. That is, thanks to the recourse of exchanging the fields by functional derivatives of the corresponding sources, and also calculating the remaining Gaussian integral, the following expression for the generating functional is obtained [24]:

$$\begin{aligned} Z\left[J,\bar{\eta},\eta,\bar{\xi},\xi\right] = N \exp\Bigg\{ i \Bigg[&\frac{S^G_{abc}}{3! i^3} \frac{\delta}{\delta J_a} \frac{\delta}{\delta J_b} \frac{\delta}{\delta J_c} + \frac{S^G_{abcd}}{4! i^4} \frac{\delta}{\delta J_a} \frac{\delta}{\delta J_b} \frac{\delta}{\delta J_c} \frac{\delta}{\delta J_d} \\ &+ \frac{S^{Gh}_{ras}}{2! i^3} \frac{\delta}{\delta \bar{\eta}_r} \frac{\delta}{\delta J_a} \frac{\delta}{\delta(-\eta_s)} + \frac{S^Q_{iaj}}{i^3} \frac{\delta}{\delta \bar{\xi}_i} \frac{\delta}{\delta J_a} \frac{\delta}{\delta(-\xi_j)} \Bigg] \Bigg\} Z_0\left[J,\bar{\eta},\eta,\bar{\xi},\xi\right], \end{aligned} \tag{2.10}$$

with

$$Z_0\left[J,\bar{\eta},\eta,\bar{\xi},\xi\right] = \exp\left\{ i \frac{J_a G_G^{ab} J_b}{2} + i \bar{\eta}_r G_{Gh}^{rs} \eta_s + i \bar{\xi}_i G_Q^{ij} \xi_j \right\}, \tag{2.11}$$

and the normalization factor is

$$\begin{aligned} N^{-1} = \Bigg[\exp\Bigg\{ i \Bigg[&\frac{S^G_{abc}}{3! i^3} \frac{\delta}{\delta J_a} \frac{\delta}{\delta J_b} \frac{\delta}{\delta J_c} + \frac{S^G_{abcd}}{4! i^4} \frac{\delta}{\delta J_a} \frac{\delta}{\delta J_b} \frac{\delta}{\delta J_c} \frac{\delta}{\delta J_d} \\ &+ \frac{S^{Gh}_{ras}}{2! i^3} \frac{\delta}{\delta \bar{\eta}_r} \frac{\delta}{\delta J_a} \frac{\delta}{\delta(-\eta_s)} + \frac{S^Q_{iaj}}{i^3} \frac{\delta}{\delta \bar{\xi}_i} \frac{\delta}{\delta J_a} \frac{\delta}{\delta(-\xi_j)} \Bigg] \Bigg\} Z_0\left[J,\bar{\eta},\eta,\bar{\xi},\xi\right] \Bigg]_{J,\bar{\eta},\eta,\bar{\xi},\xi=0}, \end{aligned}$$

where use have been made of the DeWitt compact notation [25], in which a Latin letter $(a, b, ...)$, for a field symbolizes its space-time coordinates as well as all its internal quantum numbers. The same index in a source or a tensor symbolizes the same set of variables of the kind of fields associated with this specific index. For example $A^a = A^a_{\mu_a}(x_a)$, $c^a = c^a(x_a)$ and $\Psi^i = \Psi^i(x_i)$. Such a procedure was useful for the calculations and should not create confusion. As usual, repeated indexes represent the corresponding space time integrals and contracted, Lorentz, spinor or color components.

The following definitions have been used in (2.10):

$$
\begin{aligned}
S^{\alpha}_{ijk\ldots} &\equiv \left(\frac{\delta}{\delta\Phi_i}\frac{\delta}{\delta\Phi_j}\frac{\delta}{\delta\Phi_k}\ldots S^{\alpha}\left(\Phi\right)\right)_{\Phi=0} \quad \text{for } \ \alpha = G,\ Gh,\ Q \ \text{ and } \Phi = A,\ \bar{c},\ c,\ \bar{\Psi},\ \Psi \\
G^{ij}_{\alpha} &\equiv -S^{-1}_{\alpha,ij} \qquad\qquad\qquad\qquad \text{for } \ \alpha = G,\ Gh,\ Q,
\end{aligned}
$$

where $G,\ Gh$ and Q mean the gluon, ghost and quark parts of the action respectively.

There is only one main element determining the difference of the usual Perturbative QCD expansion and the modified one considered in [13]-[15]. It is related with the form of the gluon propagator. As proposed in [13] there is an additional term in the gluon propagator which is absent in the standard expansion. In [14, 15], such a term was shown to be a consequence of a Wick expansion based in a state constructed by acting on the usual vacuum with an exponential of zero momentum pairs of gluon and ghost creation operators. By the side, the ghost propagator [14, 15] could remain unmodified if the parameter, free after fixing the form of the wave-function, is taken as a real and positive one [14, 15]. Such a selection will be employed here. In Sect. 3.2 possible implications of the ghost propagator modification will be discussed. The quark propagator is not affected in any way by the gluon condensation as introduced in [13]-[15].

From a more technical point of view the operator quantization employed in [14, 15] takes the ghost fields as satisfying the conjugation properties defined by the Kugo and Ojima approach [20, 21]. However, at the level of the Feynman diagram expansion the difference with the standard procedure is only a change of variables.

In accordance with above remarks and the results of [13]-[15] the diagram technique defining the modified expansion has the following basic elements

Propagators

$$
\begin{aligned}
G^{ab}_{G} &= \delta^{ab} g^{\mu_a\mu_b}\left[\frac{1}{k^2+i\varepsilon} - iC\delta\left(k\right)\right], \\
G^{rs}_{Gh} &= \delta^{rs}\frac{(-i)}{k^2+i\varepsilon}, \\
G^{ij}_{Q} &= \delta^{ij}\frac{m+p^{\mu}\gamma_{\mu}}{(m^2-p^2-i\varepsilon)}.
\end{aligned}
$$

vertexes

$$
\begin{aligned}
S^G_{abc} &= (2\pi)^4\,\delta\left(k_a+k_b+k_c\right)(-i)\,gf^{abc}\Big[g_{\mu_a\mu_c}\left(k^a-k^c\right)_{\mu_b}+g_{\mu_a\mu_b}\left(k^b-k^a\right)_{\mu_c} \\
&\qquad +g_{\mu_b\mu_c}\left(k^c-k^b\right)_{\mu_a}\Big], \\
S^G_{abcd} &= (2\pi)^4\,\delta\left(k_a+k_b+k_c+k_d\right)\left(-g^2\right)\Big[f^{eab}f^{ecd}\left(g_{\mu_a\mu_c}g_{\mu_b\mu_d}-g_{\mu_b\mu_c}g_{\mu_a\mu_d}\right)+ \\
&\qquad + f^{ecb}f^{ead}\left(g_{\mu_a\mu_c}g_{\mu_b\mu_d}-g_{\mu_d\mu_c}g_{\mu_a\mu_b}\right) \\
&\qquad + f^{edb}f^{eca}\left(g_{\mu_d\mu_c}g_{\mu_a\mu_b}-g_{\mu_b\mu_c}g_{\mu_a\mu_d}\right)\Big], \\
S^{Gh}_{ras} &= (2\pi)^4\,\delta\left(k_r+k_a+k_s\right)gf^{ras}\left(k^r-k^s\right)_{\mu_a}, \\
S^{Q}_{iaj} &= (2\pi)^4\,\delta\left(k_i+k_a+k_j\right)gT^a_{ij}\gamma_{\mu_a}.
\end{aligned}
$$

The propagators and vertexes will be represented by,

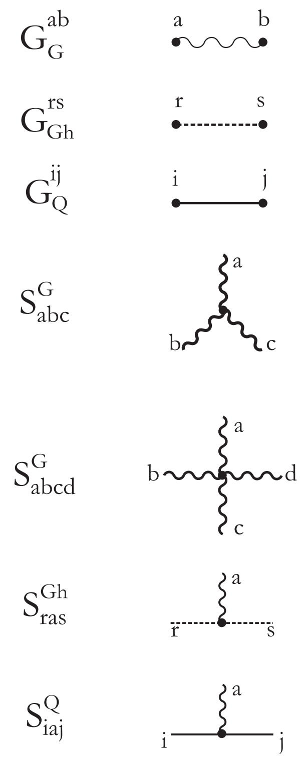


Figure 2.1: Propagator and vertex representations.

Being defined the Feynman rules under consideration, in the next section they will be applied to the calculation of various quantities of physical interest.

Chapter 3

Calculations within the Modified Perturbation Theory

Some magnitudes of physical interest are calculated within the modified perturbation theory, up to the g^2 order, and in the approximation described in the Introduction. That is disregarding all the terms not including the gluon condensate. The mean value of G^2 (Section 3.1) and the gluon mass (Section 3.2) are determined improving previous calculations [13]. In Section 3.3 the constituent quarks masses are calculated. Finally the effective potential [13] is obtained in terms of the mean value of G^2 (Section 3.4).

3.1 Mean value of G^2

The calculation of the mean value of G^2 is considered in this section. It completes the former evaluation done in [13] in the tree approximation. The result will be employed afterwards for fixing the value of the condensation parameter g^2C in order to obtain the currently accepted value for $\langle g^2G^2\rangle$. As it was stated in the introduction, the quantities will be exactly calculated disregarding all the terms not containing the influence of the condensate. It can be remarked that all the terms up to the g^2 order considered here, in which the condensate parameter enters and using dimensional regularization, do not need for renormalization. This result is another resemblance with a quasi-classical approximation.

For the calculation of $\langle G^2\rangle$ the following expression will be used

$$\langle 0|\, S_g\, |0\rangle = \frac{\int D\left(A,\bar{c},c,\bar{\Psi},\Psi\right) S_g\left[A\right] \exp\left\{i\int d^4x\left(\mathcal{L}+J^{\mu,a}A^a_\mu+\overline{c}^a\eta^a+\bar{\eta}^a c^a+\bar{\Psi}^i\xi^i+\bar{\xi}^i\Psi^i\right)\right\}}{\int D\left(A,\bar{c},c,\bar{\Psi},\Psi\right)\exp\left\{i\int d^4x\mathcal{L}\right\}}, \tag{3.1}$$

where $S_g[A]$ represents the gluon part of the action in absence of the gauge breaking term depending on the gauge parameter α, that is

$$S_g[A] = -\frac{1}{4}\int d^4x F^a_{\mu\nu}(x) F^{\mu\nu,a}(x). \tag{3.2}$$

Through the use again of the trick of changing the fields inside the functional integral by derivatives in corresponding sources, it follows for (3.1)

$$\langle 0|S_g|0\rangle = \left\{ \left[\frac{S^g_{ab}}{2i^2}\frac{\delta}{\delta J_a}\frac{\delta}{\delta J_b} + \frac{S^G_{abc}}{3!i^3}\frac{\delta}{\delta J_a}\frac{\delta}{\delta J_b}\frac{\delta}{\delta J_c} + \frac{S^G_{abcd}}{4!i^4}\frac{\delta}{\delta J_a}\frac{\delta}{\delta J_b}\frac{\delta}{\delta J_c}\frac{\delta}{\delta J_d}\right] Z\left[J,\bar{\eta},\eta,\bar{\xi},\xi\right] \right\}_{J,\bar{\eta},\eta,\bar{\xi},\xi=0} \tag{3.3}$$

where $Z\left[J,\bar{\eta},\eta,\bar{\xi},\xi\right]$ is given by the expression (2.10). In the momentum representation S^g_{ab} has the form

$$S^g_{ab} = (2\pi)^4\,\delta(k_a+k_b)\,\delta^{ab}\left[-\left(g_{\mu_a\mu_b}k_a^2 - k_{a,\mu_a}k_{a,\mu_a}\right)\right],$$

and it will be represented in the calculations by

Figure 3.1: Representation of S_{ab}.

It should be noticed that the contribution of diagrams including any kind of particles (which involve the condensate effects), to the vacuum expectation value of S_g, needs for the evaluation of loop integrals which only can be properly accounted after renormalization. However, in the present situation for these contributions to $\langle 0|S_g|0\rangle$, it happens that the loop integrals which appears are evaluated at zero momentum. Then, the results vanish using the dimensional regularization and thus do not depend on the renormalization conditions.

After computing all the diagrams appearing in the $\langle 0|S_g|0\rangle$ expression, with the remarks made above, was obtained that the only diagram contributing is the one appearing in the Figure 3.6. That is, all the diagrams not considered in earlier works have a null contribution. The details of the calculation are exposed in the Appendix A.

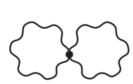
$$\langle O|S_g|O\rangle = -\frac{1}{8}$$ 

Figure 3.2: Diagram contributing to $\langle 0|S_g|0\rangle$.

The diagram in Figure 3.2 was calculated before [13, 14], the result is

$$\langle 0|\, S_g \,|0\rangle = -\frac{1}{4}\int d^4x \frac{288 g^2 C^2}{(2\pi)^8}, \tag{3.4}$$

which, in turns implies for the mean value of G^2 the result

$$\langle G^2 \rangle = \frac{288 g^2 C^2}{(2\pi)^8}. \tag{3.5}$$

Finally, making use of a currently estimated value for $\langle g^2 G^2 \rangle$:

$$\langle g^2 G^2 \rangle \cong 0.5 \left(GeV/c^2\right)^4,$$

the parameter $g^2 C$ is determined to have the value:

$$g^2 C = 64.94 \ \left(GeV/c^2\right)^2. \tag{3.6}$$

The fixing of the condensation parameter allows to investigate the physical predictions of the modified expansion in the above described approximation which does not require to introduce the renormalization.

3.2 Gluon Mass

In the present section it is exposed the calculation for the gluon mass.

The expression for the effective action of the gluon sector, in the one loop approximation or up to g^2 order [25], is

$$\Gamma^G\left[\phi_c\right] = S_G\left[\phi_c\right] + \frac{1}{2i}\ln\det G_G\left[\phi_c\right], \tag{3.7}$$

in Feynman diagrams this expression can be written in the form,

$$\Gamma\left[\Phi_c\right] = \cdot + \frac{1}{2i}$$

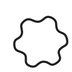


Figure 3.3: The effective action.

The second functional derivative of (3.7), in the gluon fields, defines the two point proper Green function or the inverse of the gluon propagator. It can be written in the form

$$\Gamma^G_{,ij}[\phi_c] = S^G_{,ij}[\phi_c] + \frac{1}{2i}\left(S^G_{,ipqj}[\phi_c]\,G^{pq}_G + S^G_{,ipq}[\phi_c]\,G^{qr}_G G^{ps}_G S^G_{,srj}[\phi_c]\right). \tag{3.8}$$

where the quark and ghost fields do not contribute in the approximation considered. That is, their diagrams do not involve the gluon condensate.

As it was stated in a previous work [13], the interest is centered in the case of zero mean values of the gluonic field (as it is required by the Lorentz invariance), so the expression (3.8) takes the form,

$$\Gamma^G_{,ij}[0] = S^G_{ij} + \frac{1}{2i}\left(S^G_{ipqj}G^{pq}_G + S^G_{ipq}G^{qr}_G G^{ps}_G S^G_{srj}\right), \tag{3.9}$$

or diagrammatically

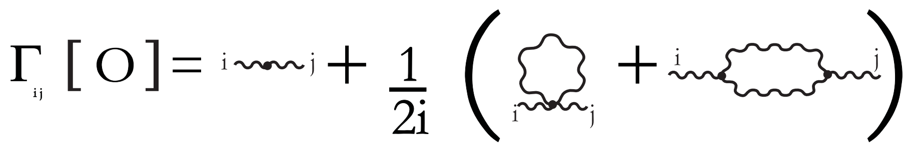


Figure 3.4: Two point proper Green function.

Writing the expressions in the momentum representation, the following results are obtained for the gluon condensate contributions

$$S^G_{ij} = -\delta^{a_i a_j} g_{\mu_i\mu_j} p^2, \tag{3.10}$$

$$S^G_{ipqj}G^{pq}_G = i\frac{6g^2CN}{(2\pi)^4}\delta^{a_i a_j} g_{\mu_i\mu_j}, \tag{3.11}$$

$$S^G_{ipq}G^{qr}_G G^{ps}_G S^G_{srj} = i\frac{g^2CN}{(2\pi)^4}\delta^{a_i a_j}\left(-10g_{\mu_i\mu_j} + 4\frac{p_{\mu_i}p_{\mu_j}}{p^2}\right). \tag{3.12}$$

The sum of the three contributions is

$$\Gamma^G_{,ij}[0;p] = \delta^{a_i a_j}\left[-\left(p^2 + \frac{2g^2CN}{(2\pi)^4}\right)\left(g_{\mu_i\mu_j} - \frac{p_{\mu_i}p_{\mu_j}}{p^2}\right) - p^2\frac{p_{\mu_i}p_{\mu_j}}{p^2}\right]. \tag{3.13}$$

For calculating the previous expressions the following relation, between the structure constants for a gauge group $SU(N)$, was employed

$$f^{a_i a_l a_k} f^{a_j a_l a_k} = N \delta^{a_i a_j}. \tag{3.14}$$

Considering the specific group $SU(3)$, the expression (3.13) takes the form

$$\Gamma^G_{,ij}[0;p] = \delta^{a_i a_j} \left[-\left(p^2 + \frac{6g^2 C}{(2\pi)^4}\right)\left(g_{\mu_i \mu_j} - \frac{p_{\mu_i} p_{\mu_j}}{p^2}\right) - p^2 \frac{p_{\mu_i} p_{\mu_j}}{p^2} \right]. \tag{3.15}$$

The eigenvalues of the previous expression define the dispersion relations, and have the form

$$p^2 = 0, \quad p^2 - m_G^2 = 0, \tag{3.16}$$

where

$$m_G^2 = -\frac{6g^2 C}{(2\pi)^4} = -0.25 \left(GeV/c^2\right)^2. \tag{3.17}$$

Therefore, as the parameter C was defined before as real and positive, in the framework of the initial state considered [14, 15], it follows that the transverse gluon mass correction becomes tachyonic. The ability of a tachyonic mass in producing improvements in models for the inter quark potential has been recently argued in the literature [26, 27]. The effect is related with the introduction of a linearly rising term as a first correction to the Coulomb potential in the massless case [27]. Therefore, the tachyonic result arising here for the gluon mass, appears to be of interest for an attempt of deriving from the present framework the already existing successful phenomenological bound state models for mesons [28]-[30].

Finally it should be mentioned that if a modification of the ghost propagator is considered, then appears a longitudinal mass term for the propagator, as consequence of the diagram in Fig. 3.5 and the polarization operator (gluon self-energy) becomes non-transverse. This result means a violation of a Ward identity representing the gauge invariance, by the presence of the ghost condensate. Thus, it follows from the present results that the selection of the parameter for the initial state in [14, 15] was the appropriate one for the fulfillment of a gauge invariance condition at this stage. The physical meaning of alternative procedures for constructing the initial state need however a clarification.

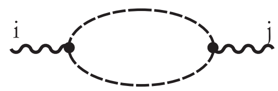


Figure 3.5: Ghost contribution to the gluon self-energy.

3.3 Quark Masses

The effects produced by the condensate, on the quark masses, are considered in this section. The full inverse propagator for quarks, in terms of the corresponding mass operator, can be expressed as [31]

$$G_Q^{-1}(p) = \left(m_Q - p^\mu \gamma_\mu - \Sigma(p)\right), \tag{3.18}$$

where the color index identity matrix has been omitted and $\Sigma(p)$ is the mass operator. Its expression is represented by the diagram in Figure 3.6.

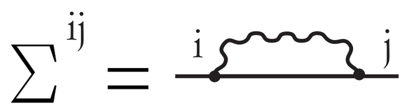


Figure 3.6: Diagram for $\Sigma(p)$.

The exact form of $\Sigma(p)$ up to the g^2 order, considering the modified Feynman rules, is

$$\Sigma(p) = g^2 C_F \int \frac{d^4k}{(2\pi)^4 i} \frac{\gamma_\mu \left(m_Q + (p-k)^\alpha \gamma_\alpha\right) \gamma_\nu G_G^{\mu\nu}(k)}{\left(m_Q^2 - (p-k)^2\right)}, \tag{3.19}$$

in which the algebra for the color indexes has been calculated [31], leading to the following expression for the constant C_F $(SU(N))$

$$C_F = \frac{N^2 - 1}{2N},$$

for the case of interest $SU(3)$, it reduces to $C_F = \frac{4}{3}$.

Taking into account the modified gluon propagator and the standard relations for the γ matrices (which here are adopted the same as in Ref. [31]) the full one loop mass operator expression can be simplified to

$$\Sigma\left(p\right)=g^{2}C_{F}\int\frac{d^{4}k}{\left(2\pi\right)^{4}i}\frac{2\left(2m_{Q}-\left(p-k\right)^{\alpha}\gamma_{\alpha}\right)}{\left(m_{Q}^{2}-\left(p-k\right)^{2}\right)}\left(\frac{1}{k^{2}}-iC\delta\left(k\right)\right). \qquad (3.20)$$

Here, as before, the term not involving the condensate won't be considered. In this case the expression for $\Sigma\left(p\right)$ reduces to

$$\Sigma\left(p\right)=-\frac{g^{2}C_{F}C}{\left(2\pi\right)^{4}}\frac{2\left(2m_{Q}-p^{\alpha}\gamma_{\alpha}\right)}{\left(m_{Q}^{2}-p^{2}\right)}. \qquad (3.21)$$

Then the expression for the inverse quark propagator reduces to

$$G_{Q}^{-1}\left(p\right)=m_{Q}\left(1+2\frac{M^{2}}{\left(m_{Q}^{2}-p^{2}\right)}\right)-p^{\mu}\gamma_{\mu}\left(1+\frac{M^{2}}{\left(m_{Q}^{2}-p^{2}\right)}\right), \qquad (3.22)$$

where it is defined

$$M^{2}=\frac{2g^{2}C_{F}C}{\left(2\pi\right)^{4}}=0.11\left(GeV/c^{2}\right)^{2},$$

in which the numerical value has been obtained considering (3.6).

The zeros of the determinant associated to the inverse propagator (3.22) allow to determine the effects of the condensate (reflected by the parameter M), on the effective mass or mass shell of quarks. Now is useful to introduce an index $s, s=1...6$ for each kind of quark characterized by its particular current mass m_{Q_s}.

The mass shell obtained in the present case is

$$p^{2}-m_{q_{s,i}}^{2}=0, \qquad (3.23)$$

for $m_{q_{s,i}}^{2}$ are obtained three different analytical expressions arising from the solutions of a cubic equation. They are

$$\begin{aligned}
m_{q_{s,1}}^{2} &= A^{\frac{1}{3}}-B+m_{Q_{s}}^{2}+\frac{2}{3}M^{2},\\
m_{q_{s,2}}^{2} &= -\frac{1}{2}A^{\frac{1}{3}}+\frac{1}{2}B+m_{Q_{s}}^{2}+\frac{2}{3}M^{2}+\frac{1}{2}i\sqrt{3}\left(A^{\frac{1}{3}}+B\right),\\
m_{q_{s,3}}^{2} &= -\frac{1}{2}A^{\frac{1}{3}}+\frac{1}{2}B+m_{Q_{s}}^{2}+\frac{2}{3}M^{2}-\frac{1}{2}i\sqrt{3}\left(A^{\frac{1}{3}}+B\right),
\end{aligned}$$

in which A, B are given by the following expressions

$$\begin{aligned} A &= \frac{5}{6}m_{Q_s}^2 M^4 - \frac{1}{27}M^6 + \frac{1}{18}\sqrt{96m_{Q_s}^6 M^6 + 177m_{Q_s}^4 M^8 - 12m_{Q_s}^2 M^{10}}, \\ B &= \frac{\frac{2}{3}m_{Q_s}^2 M^2 - \frac{1}{9}M^4}{A^{\frac{1}{3}}}. \end{aligned}$$

In order to find the dependence of the effective quark masses $m_{q_{s,i}}^2$ as functions of the current mass parameters m_{Q_s}, an analysis for the various dispersion relations that appeared, was done. It follows that there is only one solution having a squared mass being positive for arbitrary values of the current quark masses. The existence of this real and positive solution for the squared mass is possible because of the real and positive character of C, as was proved previously [14, 15] and fixed through $\langle g^2 G^2 \rangle$. The other solutions for the masses become complex for certain values of the current masses m_{Q_s}, and there is no a clear understanding of their meanings.

For the purely real solution the value of the effective quark masses $m_{q_s} = f\left(m_{Q_s}\right)$ as functions of m_{Q_s} is shown in the Figure 3.7. The graph is plotted for the region $m_{Q_s} < 2\,GeV/c^2$ which contains the current mass values of the $u,\ d,\ s$ and c quarks.

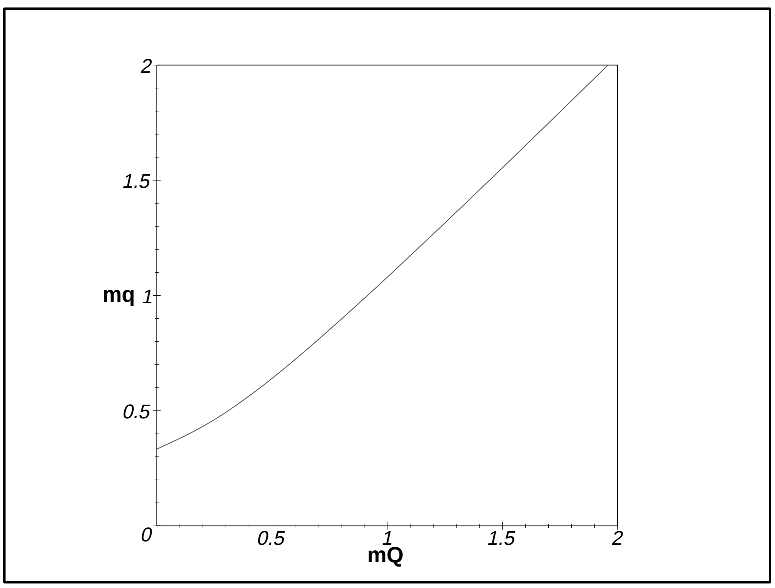


Figure 3.7: Real solution for the quark mass as a function of the Lagrangian mass (masses in GeV/c^2).

As it can be appreciated in the picture, the calculated effective quark masses for light flavors (u, d and s) are clearly predicting the values of the quark masses being in use in the constituent quark models of hadrons. That is, the light quarks get a weight of near one third of the nucleon mass. In table 3.1 the mass values obtained, for each quark flavor, are shown. The reported lower and upper values for the current masses are denoted by m_{Low}^{Exp} and m_{Up}^{Exp}, respectively [11]. The correspondingly calculated constituent masses are indicated by m_{Low}^{Th} and m_{Up}^{Th}; finally m_{Med}^{Th} is the evaluated mean value of the constituent masses.

Table 3.1: Quark mass values (MeV/c^2) in presence of the condensate.

$Quarks$	m_{Low}^{Exp}	m_{Up}^{Exp}	m_{Low}^{Th}	m_{Up}^{Th}	m_{Med}^{Th}
Up (u)	1.5	5	334.1	335.8	334.9
Down (d)	3	9	334.8	337.8	336.3
Strange (s)	60	170	361.3	415.1	388.2
Charmed (c)	1100	1400	1172.6	1457.9	1315.2
Bottom (b)	4100	4400	4120.3	4418.9	4269.6
Top (t)	168600	179000	168600.5	179000.5	173800.5

From the global properties of table 3.1 it can be observed that the main effect of the gluon condensate seems to be dressing the light quarks with a cloud of gluons having a total mass of one third of the nucleon mass. Then, the results point in the direction of the mainly glue nature of the constituent quark masses of the u, d and s quarks within many baryon resonance. These results support the idea that a modified perturbative expansion like the one being considered, in which the effect of the gluon condensate has been incorporated, would be able to predict with reasonably good approximation properties of the low energy strong interactions. Similar values for the constituent quarks masses have been obtained by a different method in [32].

In tables 3.2 and 3.3 are shown the masses for baryon and meson ground states [11], compared with the values obtained for them by a theoretical estimate obtained as the simple addition of the calculated constituent masses for each of their known constituent quarks.

Table 3.2: Experimental and theoretical masses (MeV/c^2) for baryonic resonances in their ground states.

Baryon	Exp.Val.	Th.Val.	Rel.Err.
p(uud)	938.27231	1006.2	7.2
n(udd)	939.56563	1007.5	7.2
Λ(uds)	1115.683	1059.4	5.0
Σ^+(uus)	1189.37	1058.1	11.0
Σ^0(uds)	1192.642	1059.4	11.2
Σ^-(dds)	1197.449	1060.8	11.4
Ξ^0(uss)	1314.9	1111.4	15.5
Ξ^-(dss)	1321.32	1112.7	15.8
Ω^-(sss)	1642.45	1164.6	29.1
Λ_c^+(udc)	2284.9	1986.5	13.1
Ξ_c^+(usc)	2465.6	2038.4	17.3
Ξ_c^0(dsc)	2470.3	2039.7	17.4
Ω_c^0(ssc)	2704	2091.6	22.6
Λ_b^0(udb)	5624	4940.8	12.2

Table 3.3: Experimental and theoretical masses (MeV/c^2) for vector messon resonances in their ground states.

Meson	Exp.Val.	Th.Val.	Rel.Err.
$\rho\left(\frac{u\overline{u}-d\overline{d}}{\sqrt{2}}\right)$	770.0	671.2	12.8
$\varpi\left(\frac{u\overline{u}+d\overline{d}}{\sqrt{2}}\right)$	781.94	671.2	14.2
$\phi\,(s\overline{s})$	1019.413	776.4	23.8
$J/\psi\,(1S)\,(c\overline{c})$	3096.88	2630.5	15.1
$Y\,(1S)\,\left(b\overline{b}\right)$	9460.37	8539.1	9.7

As it can be appreciated the results obtained for the vector meson and baryon resonances are only estimates. However, their values are in good correspondence with the reported experimental values. This situation is more curious taking into account: 1) The great errors in the quark current masses [11] 2) The standard one loop self energy contribution to the propagator was not included. The major differences between the experimental values and the theoretical calculations are obtained for resonances containing the strange quark. The improvement of these results could be introduced from higher order corrections, because the strange quark is located between the almost massless up, down and the heavy quarks, then the one loop correction could not be enough to obtain the total influence of the condensate on that quark. It can be noticed that the validity of the results obtained implies that the binding energy contribution to the baryon rest masses is small. This property then could justify a no relativistic description of such resonances.

3.4 Effective Potential

In the present section is calculated the effective potential as a function of the mean value of G^2. The approximation considered is the same advanced in a previous work [13], and the result is improved through the relation between the mean value of G^2 and the gluon mass obtained in previous sections. It is shown that the condensate is spontaneously generate from the standard vacuum, as in the Savvidy models [1].

The expression for the effective potential, considering the presence of a mean field, was obtained by R. Jackiw [33] and has the form

$$V\left(\phi_c\right) = V_0\left(\phi_c\right) + \frac{1}{2i}\int \frac{d^4k}{\left(2\pi\right)^4} \ln\det\left(-S_{,ij}\left[\phi_c;k\right]\right) + i\left\langle \exp\left(i\int d^4x\mathcal{L}_i\left[\phi_c,\phi\right]\right)\right\rangle, \qquad (3.24)$$

as it was mentioned before, in the present work is analyzed the case for null mean values of gluonic fields ($\phi_c = 0$), as it is required by the Lorentz invariance. The first term in the previous expression is $\langle G^2\rangle/4$, the second term represents the quantum corrections to the effective potential up to one loop approximation, and the last term represents the higher order corrections.

The approximation considered in the present section [13], is the one in which all the mass insertions, consequence of the gluon condensate, are introduced in the one loop term of the expression (3.24). That is, it is substituted the $S^G_{,ij}\left[0;k\right]$ by the $\Gamma^G_{,ij}\left[0;k\right]$ calculated in Section 3.2, and it is like considering $\Gamma^G_{,ij}\left[0;k\right]$ as a modified tree approximation for the second derivative of the action under the influence of the condensate. These corrections appear in an exact calculation from the expansion of the third term of (3.24), that won't be taken into account in what follows. In this case the expression (3.24) takes the form

$$V\left(\langle G^2\rangle\right) = \frac{\langle G^2\rangle}{4} + \frac{1}{2i}\int \frac{d^4k}{\left(2\pi\right)^4} \ln\det\left(-\Gamma^G_{ij}\left[0;k\right]\right), \qquad (3.25)$$

that considering the relation

$$\ln\det\left(-\Gamma^{G}_{ij}\left[0;k\right]\right)=\mathrm{Tr}\ln\left(-\Gamma^{G}_{ij}\left[0;k\right]\right), \tag{3.26}$$

can be written as

$$V\left(\langle G^2\rangle\right)=\frac{\langle G^2\rangle}{4}+\frac{1}{2i}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\ln\left(-\Gamma^{G}_{ij}\left[0;k\right]\right), \tag{3.27}$$

In order to calculate the previous integral, the relation (3.15) can be written in the form

$$\Gamma^{G}_{,ij}\left[0;k\right]=-\delta^{a_ia_j}\left[\left(k^2-m_G^2\right)P_T+k^2P_L\right], \tag{3.28}$$

where P_T is the transverse projection operator

$$P_T=g_{\mu_i\mu_j}-\frac{k_{\mu_i}k_{\mu_j}}{k^2}, \tag{3.29}$$

and P_L is the longitudinal projection operator

$$P_L=\frac{k_{\mu_i}k_{\mu_j}}{k^2}. \tag{3.30}$$

These operators satisfy the following relations:

$$(P_T)^N = P_T, \quad \text{for } N>0 \tag{3.31}$$

$$(P_L)^N = P_L, \quad \text{for } N>0 \tag{3.32}$$

$$P_T\cdot P_L = 0, \tag{3.33}$$

$$\mathrm{Tr}(\delta^{a_ia_j}P_T) = 8\times 3=24, \tag{3.34}$$

$$\mathrm{Tr}(\delta^{a_ia_j}P_L) = 8\times 1=8. \tag{3.35}$$

Then it is possible to write

$$\begin{aligned}\mathrm{Tr}\ln\left(-\Gamma^{G}_{ij}\left[0;k\right]\right) &= \mathrm{Tr}\ln\left(\delta^{a_ia_j}\left[\left(k^2-m_G^2\right)P_T+k^2P_L\right]\right)\\ &= 24\ln\left(k^2-m_G^2\right)+8\ln\left(k^2\right),\end{aligned} \tag{3.36}$$

and considering that

$$\int\frac{d^4k}{(2\pi)^4 i}\ln\left(k^2\right)=const, \tag{3.37}$$

the attention will be centered in the term

$$\int\frac{d^4k}{(2\pi)^4 i}\ln\left(k^2-m_G^2\right)=\int\frac{d^4k}{(2\pi)^4 i}\ln\left(1-\frac{m_G^2}{k^2}\right)+const. \tag{3.38}$$

For the calculation is defined:

$$T_m = \int \frac{d^4k}{(2\pi)^4\, i} \ln\left(1 - \frac{m_G^2}{k^2}\right). \tag{3.39}$$

It is not easy calculate the integral in the previous expression, because it should be compute in the Minkowski space. The solution of this problem is obtained through a 90° rotation in the complex plane k_0, with this transformation the Minkowski integration is replaced by an Euclidean integration. Such a rotation is called Wick rotation and is equivalent to a change of variables

$$k_0 = iK_{0,} \qquad \text{with} \quad K_0 \text{ real} \tag{3.40}$$

then

$$d^4k = id^4K, \qquad k^2 = -K^2, \quad \text{with} \;\; K^2 = K_0^2 + \overrightarrow{K}^2. \tag{3.41}$$

and (3.39) takes the form

$$T_m = \int \frac{d^4K}{(2\pi)^4} \ln\left(1 + \frac{m_G^2}{K^2}\right). \tag{3.42}$$

Differentiating the previous expression in m_G^2 it is possible to eliminate the logarithm in the integral

$$\frac{dT_m}{dm_G^2} = \int \frac{d^4K}{(2\pi)^4} \frac{1}{K^2 + m_G^2} = \int_0^\infty \frac{K^3 dK}{K^2 + m_G^2} \int \frac{d\Omega}{(2\pi)^4}, \tag{3.43}$$

and considering for the angular integration the expression [31]:

$$\int \frac{d\Omega}{(2\pi)^4} = \frac{2\pi^2}{(2\pi)^4}, \tag{3.44}$$

it is possible to rewrite (3.43) in the form

$$\frac{dT_m}{dm_G^2} = \frac{2\pi^2}{(2\pi)^4} \int_0^\infty \frac{K^3 dK}{K^2 + m_G^2}. \tag{3.45}$$

The momentum integral in (3.45) is divergent, so it is needed to use dimensional regularization to calculate it. The area of a D-dimensions unitary sphere is

$$S_{D-1} = \frac{2\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})}, \tag{3.46}$$

then the integral (3.45) in D-dimensions has the form [25]

$$\frac{dT_m}{dm_G^2}=\frac{S_{D-1}}{(2\pi)^D}\int\limits_0^\infty\frac{K^{D-1}dK}{K^2+m_G^2}=\frac{m_G^{2-\varepsilon}}{(4\pi)^{2-\varepsilon}}\Gamma\left(-1+\frac{\varepsilon}{2}\right) \tag{3.47}$$

where $\varepsilon=4-D$ and in general

$$\begin{aligned}\Gamma\left(-n+\frac{\varepsilon}{2}\right) &= \frac{(-1)^n}{n!}\left[\frac{1}{\varepsilon}+\Psi\left(n+1\right)+O\left(\varepsilon\right)\right],\\ \Psi\left(n+1\right) &= 1+\frac{1}{2}+...+\frac{1}{n}-\gamma,\\ \Psi\left(1\right) &= -\gamma \quad \text{with} \;\; \gamma=0.5772\end{aligned} \tag{3.48}$$

All the functions in (3.47) are expanded for $\varepsilon\to 0$, and cutting the pole in ε the expression (3.47) takes the form,

$$\frac{dT_m}{dm_G^2}=\left(\frac{m_G}{4\pi}\right)^2\left(\ln\left(\frac{m_G}{4\pi}\right)^2-const\right), \tag{3.49}$$

and the result for the integral in m_G^2 is

$$T_m=\frac{1}{64\pi^2}m_G^4\ln\frac{m_G^4}{const}. \tag{3.50}$$

Finally, the expression (3.24) can be written as

$$V\left(\langle G^2\rangle\right)=\frac{\langle G^2\rangle}{4}+12\frac{1}{64\pi^2}m_G^4\ln\frac{m_G^4}{const}, \tag{3.51}$$

that considering (3.17) and (3.5) takes the form

$$V\left(\langle G^2\rangle\right)=\frac{\langle G^2\rangle}{4}+\frac{3}{128}g^2\langle G^2\rangle\ln\frac{g^2\langle G^2\rangle}{const}. \tag{3.52}$$

which has a minimum for G^2 different from zero. This implies that the gluon condensate is generated spontaneously from the zero condensate vacuum, as for Savvidy models.

Chapter 4

Generalization of the Theory for arbitrary values of the α parameter

The generalization of the proposed theory, for arbitrary values of the gauge parameter α, is explored. In Sec. 4.1, the ansatz for the Fock space state that generates the desired perturbative expansion is defined. The proof that the state satisfies the physical state conditions imposed for the BRST formalism is also analyzed in this section. In Sec. 4.2 the modifications to the usual perturbation theory are calculated, and the introduced parameters fixed in such a way of making explicit the Lorentz invariance in the propagator and reproducing a previous proposition [13].

4.1 Modified Initial State

In the present section it is considered the physical vacuum state able to modify the usual perturbative theory in the way proposed previously [13], for arbitrary values of the α parameter. Some needed results of the Kugo and Ojima quantization, for arbitrary values of the gauge parameter, are also pointed out.

The equations of motion for arbitrary values of α, deduced from the Kugo and Ojima Lagrangian [20, 21], are

$$\begin{aligned} \Box A_{\mu}^{a}\left(x\right)-\left(1-\alpha\right)\partial_{\mu}B^{a}\left(x\right) &= 0, \\ \partial^{\mu}A_{\mu}^{a}\left(x\right)+\alpha B^{a}\left(x\right) &= 0, \\ \Box B^{a}\left(x\right)=\Box c^{a}\left(x\right)=\Box\overline{c}^{a}\left(x\right) &= 0. \end{aligned} \tag{4.1}$$

Only the solution obtained for the gluonic field differs from the one obtained for $\alpha = 1$, considered earlier [14, 15], and takes the form [20, 21]

$$A^a_\mu(x) = \sum_{\vec{k}} \left(\sum_{\sigma=1,2} A^a_{\vec{k},\sigma} f^\sigma_{k,\mu}(x) + A^{L,a}_{\vec{k}} f_{k,L,\mu}(x) + B^a_{\vec{k}} \left[f_{k,S,\mu}(x) + (1-\alpha)\partial_\mu D^{(1/2)}_{(x)} g_k(x) \right] \right) + h.c., \tag{4.2}$$

where the wave packets in (4.2) are taken as

$$\begin{aligned} g_k(x) &= \frac{1}{\sqrt{2Vk_0}} \exp(-ikx), \quad kx = k_0x_0 - \vec{k}\vec{x}, \quad k_0 = \left|\vec{k}\right| \\ f^\sigma_{k,\mu}(x) &= \frac{1}{\sqrt{2Vk_0}} \epsilon^\sigma_\mu(k) \exp(-ikx). \end{aligned} \tag{4.3}$$

The polarization vectors $\epsilon^\sigma_\mu(k)$, $\epsilon_{L,\mu}(k)$, $\epsilon_{S,\mu}(k)$ are defined by

$$\begin{aligned} \vec{k}\cdot\vec{\epsilon^\sigma}(k) &= 0, \ \epsilon^\sigma_0(k) = 0, \ \vec{\epsilon^\sigma}(k)\cdot\vec{\epsilon^\tau}(k) = \delta^{\sigma\tau}, \ \text{for } \sigma,\tau = 1,2 \\ \epsilon_{L,\mu}(k) &= -ik_\mu = -i\left(\left|\vec{k}\right|, -\vec{k}\right), \\ \epsilon_{S,\mu}(k) &= -i\frac{\overline{k}_\mu}{2\left|\vec{k}\right|^2} = \frac{-i\left(\left|\vec{k}\right|, \vec{k}\right)}{2\left|\vec{k}\right|^2}. \end{aligned} \tag{4.4}$$

The commutation relations between the creation and annihilation operators are given by

$$\begin{array}{cccccc} & A^{a'+}_{\vec{k}',\sigma'} & A^{L,a'+}_{\vec{k}'} & B^{a'+}_{\vec{k}'} & c^{a+}_{\vec{k}'} & \overline{c}^{a+}_{\vec{k}'} \\ A^a_{\vec{k},\sigma} & \delta^{aa'}\delta_{\vec{k}\vec{k}'}\delta_{\sigma\sigma'} & 0 & 0 & 0 & 0 \\ A^{L,a}_{\vec{k}} & 0 & 0 & -\delta^{aa'}\delta_{\vec{k}\vec{k}'} & 0 & 0 \\ B^a_{\vec{k}} & 0 & -\delta^{aa'}\delta_{\vec{k}\vec{k}'} & 0 & 0 & 0 \\ c^a_{\vec{k}} & 0 & 0 & 0 & 0 & i\delta^{aa'}\delta_{\vec{k}\vec{k}'} \\ \overline{c}^a_{\vec{k}} & 0 & 0 & 0 & -i\delta^{aa'}\delta_{\vec{k}\vec{k}'} & 0 \end{array} \tag{4.5}$$

The integro-differential operator $D^{(1/2)}_{(x)}$, is defined by

$$D^{(1/2)}_{(x)} \equiv 1/2(\nabla^2)^{-1}(x_0\partial_0 - 1/2), \tag{4.6}$$

and works as an "inverse" of the d'Alembertian for simple pole functions [34]

$$\Box D^{(1/2)}_{(x)} f(x) = f(x) \quad \text{if} \quad \Box f(x) = 0. \tag{4.7}$$

The value of $\partial_\mu D_{(x)}^{(1/2)} g_k(x)$ in (4.2) is determined with the use of the wave packets (4.3) and polarization vectors (4.4) considered. The result obtained is

$$\begin{aligned}\partial_\mu D_{(x)}^{(1/2)} g_k(x) &= \partial_\mu 1/2(\nabla^2)^{-1}(x_0\partial_0 - 1/2)\frac{1}{\sqrt{2Vk_0}}\exp(-ikx)\\ &= A(k_0,x_0)\, f_{k,L,\mu}(x) + B_\mu(k,x)\,,\end{aligned} \tag{4.8}$$

where $A(k_0,x_0)$ and $B_\mu(k,x)$ are defined by:

$$\begin{aligned}A(k_0,x_0) &\equiv \frac{1}{2\left|\vec{k}\right|^2}(ik_0x_0 + 1/2)\,,\\ B_\mu(k,x) &\equiv \frac{ik_0\delta_{\mu 0}}{2\left|\vec{k}\right|^2} g_k(x).\end{aligned}$$

In similar way for $\partial_\mu D_{(x)}^{(1/2)} g_k^*(x)$ is obtained:

$$\partial_\mu D_{(x)}^{(1/2)} g_k^*(x) = A^*(k_0,x_0)\, f_{k,L,\mu}^*(x) + B_\mu^*(k,x)\,. \tag{4.9}$$

Now it is possible to define the proposition for the QCD vacuum state. The considered one is a generalization of the previously proposed [14, 15], in which the constant accompanying the non-physical sector has allowed to be gauge dependent in order to obtain a covariant gluon propagator. This state is physically equivalent to the one proposed in the previous work [14, 15], because its physical part (transverse gluons part) was not modified. The modifications were introduced only in the non-physical sector, as usual for this sector, to make explicit the covariance.

The new vacuum state is defined by the expression

$$\mid\Psi\rangle = \exp\left\{\sum_{a=1}^{8}\sum_{\vec{p}_i,|\vec{p}_i|<P}\left[\sum_{\sigma=1,2}\frac{C_\sigma(P)}{2}A_{\vec{p}_i,\sigma}^{a+}A_{\vec{p}_i,\sigma}^{a+} + C_3(\alpha,P)\left(B_{\vec{p}_i}^{a+}A_{\vec{p}_i}^{L,a+} + i\overline{c}_{\vec{p}_i}^{a+}c_{\vec{p}_i}^{a+}\right)\right]\right\}\mid 0\rangle. \tag{4.10}$$

In this state the gluon pairs are created with very small momenta $\vec{p}_i$, but different from zero, with the objective of avoiding the difficulties appearing if they are considered with zero momentum directly. The sum over $\vec{p}_i$ for $|\vec{p}_i| < P$ is introduced to avoid any preferential direction in the space when the limit $P \to 0$ is considered, taking into account that the volume $V \to \infty$. Is interesting to note that this sum was not introduced in a previous work [14, 15], for the

$\alpha = 1$ analysis, because the operator $D_{(x)}^{(1/2)}$ was not involved and the results were independent from the introduced auxiliary momentum direction.

In the analysis following the work will be explicitly done with a generic color and a generic momentum $\vec{p}_i$, for simplifying the exposition. This can be done because in the free theory all different colors and momenta can be worked out independently, thanks to the commutation relations defined (4.5). At the necessary point of the analysis all the contributions will be included.

The proof that the proposed state (4.10) satisfies the required physical state conditions,

$$\begin{aligned} Q_B \mid \Psi\rangle &= 0, \\ Q_C \mid \Psi\rangle &= 0, \end{aligned} \tag{4.11}$$

is exactly the same advanced in the previous work, because the constant $C_3(\alpha, P)$ does not make any change. So this state is a physical state of the theory and it is possible to state that the non-physical sector of $\mid \Psi\rangle$ is undetectable in the physical world and it can be neglect in physical calculations, like in the calculation of the norm for example.

Then for the calculation of the norm a similar result, to the one in [14, 15], is obtained

$$N = \langle \Psi \mid \Psi \rangle = \prod_{\sigma=1,2} \prod_{a=1}^{8} \prod_{\vec{p}_i, |\vec{p}_i| < P} \left[\sum_{m=0}^{\infty} |C_\sigma(P)|^{2m} \frac{(2m)!}{(m!)^2} \right], \quad \text{for } |C_\sigma(P)| < 1$$

The normalized physical vacuum state is then defined by

$$\mid \tilde{\Psi}\rangle = \frac{1}{\sqrt{N}} \mid \Psi\rangle. \tag{4.12}$$

4.2 Modified Perturbation Theory

In the present section the modification to the usual perturbation theory is calculated. It is introduced through the modification of the usual vacuum state (empty of the Fock space), by the state proposed in the previous section (4.10). The analysis is generalized for arbitrary values of the gauge parameter α. At the end a brief analysis of the α generalization consequences, on the previous calculations for $\langle G^2 \rangle$, quark and gluon masses, is done.

As it was pointed out in a previous work [14, 15], the modification to the gluon generating functional of the free-particle Green functions (and to the propagator also) is determined by:

$$\langle\widetilde{\Psi}\mid \exp\left\{i\int d^4x \sum_{a=1,..8} J^{\mu,a}(x)\, A_\mu^{a-}(x)\right\}\exp\left\{i\int d^4x \sum_{a=1,..8} J^{\mu,a}(x)\, A_\mu^{a+}(x)\right\}\mid\widetilde{\Psi}\rangle. \quad (4.13)$$

Considering the expressions (4.2), (4.8) and (4.9), the gluon annihilation and creation operators in (4.13) have the form

$$\begin{aligned}
A_\mu^{a+}(x) &= \sum_{\vec{k}}\left(\sum_{\sigma=1,2} A^a_{\vec{k},\sigma} f^\sigma_{k,\mu}(x) + A^{L,a}_{\vec{k}} f_{k,L,\mu}(x) + \right.\\
&\left. + B^a_{\vec{k}}\left[f_{k,S,\mu}(x) + (1-\alpha)\left(A(k_0,x_0)\, f_{k,L,\mu}(x) + B_\mu(k,x)\right)\right]\right),\\
A_\mu^{a-}(x) &= \sum_{\vec{k}}\left(\sum_{\sigma=1,2} A^{a+}_{\vec{k},\sigma} f^{\sigma*}_{k,\mu}(x) + A^{L,a+}_{\vec{k}} f^*_{k,L,\mu}(x) + .\right.\\
&\left. + B^{a+}_{\vec{k}}\left[f^*_{k,S,\mu}(x) + (1-\alpha)\left(A^*(k_0,x_0)\, f^*_{k,L,\mu}(x) + B^*_\mu(k,x)\right)\right]\right). \quad (4.14)
\end{aligned}$$

As it can be noticed in the expressions (4.10) and (4.14) the transverse and ghost terms are not modified with the generalization of the theory for arbitrary values of α. Then their contributions are the same calculated previously [14, 15]. It is important to remark that the calculations are done decomposing the expression (4.13) in products of each mode contribution, thanks to commutation relations (4.5).

The result obtained for the transverse mode contributions, for $|C_\sigma(P)| < 1$, is [14, 15]

$$\begin{aligned}
&\langle 0\mid \exp\left(\sum_{\sigma=1,2}\frac{C^*_\sigma(P)}{2} A^a_{\vec{p}_i,\sigma} A^a_{\vec{p}_i,\sigma}\right)\exp\left\{i\int d^4x J^{\mu,a}(x)\sum_{\sigma=1,2} A^{a+}_{\vec{p}_i,\sigma} f^{\sigma*}_{p_i,\mu}(x)\right\}\\
&\times\exp\left\{i\int d^4x J^{\mu,a}(x)\sum_{\sigma=1,2} A^a_{\vec{p}_i,\sigma} f^\sigma_{p_i,\mu}(x)\right\}\exp\left(\sum_{\sigma=1,2}\frac{C_\sigma(P)}{2} A^{a+}_{\vec{p}_i,\sigma} A^{a+}_{\vec{p}_i,\sigma}\right)\mid 0\rangle\\
&= \exp\left\{-\sum_{\sigma=1,2}\left(J^a_{p_i,\sigma}\right)^2 \frac{\left(C_\sigma(P) + C^*_\sigma(P) + 2\left|C_\sigma(P)\right|^2\right)}{2\left(1-\left|C_\sigma(P)\right|^2\right)}\right\}, \quad (4.15)
\end{aligned}$$

where the normalization factor has been canceled, and it should be reminded that the momenta $\vec{p_i}$ are very small and the sources are located in a space finite region. The following simplified notation was also introduced [14, 15]

$$J^a_{p_i,\sigma} = \int \frac{d^4x}{\sqrt{2Vp_{i0}}} J^{\mu,a}(x)\,\epsilon_{\sigma,\mu}(p_i)\,.$$

The calculation of longitudinal and scalar mode contributions is in this occasion more elaborated, and a brief exposition of it could be found in the Appendix B, the result obtained for $|C_3(\alpha,P)|<1$ is

$$\exp\left\{-\int\frac{d^4xd^4y}{2Vp_{i0}}J^{\mu,a}(x)\,J^{\nu,a}(y)\left[\left(\frac{C_3(\alpha,P)+C_3^*(\alpha,P)+2\,|C_3(\alpha,P)|^2}{\left(1-|C_3(\alpha,P)|^2\right)}\right)\times\right.\right.$$
$$\left.\left.\times\left(\epsilon_{S,\mu}(p_i)\,\epsilon_{L,\nu}(p_i)+\frac{(1-\alpha)}{4\,|\vec{p}_i|^2}\left[\epsilon_{L,\mu}(p_i)+i2p_{i0}\delta_{\mu 0}\right]\epsilon_{L,\nu}(p_i)\right)\right]\right\},\qquad(4.16)$$

the normalization factor has been canceled.

Now substituting the expressions (4.15) and (4.16) in (4.13). Performing some algebraic manipulations, keeping in mind the properties of the polarization vectors defined (4.4), considering $C_1(P)=C_2(P)$, and finally introducing the contributions for all momenta $\vec{p}_i$ for $|\vec{p}_i|<P$, the following result is obtained

$$\exp\left\{\int\frac{d^4xd^4y}{2V}J^{\mu,a}(x)\,J^{\nu,a}(y)\sum_{\vec{p}_i,|\vec{p}_i|<P}\frac{1}{p_{i0}}\left[\left(\frac{C_1(P)+C_1^*(P)+2\,|C_1(P)|^2}{2\left(1-|C_1(P)|^2\right)}\right)g_{\mu\nu}+\right.\right.$$
$$+\left(\frac{C_3(\alpha,P)+C_3^*(\alpha,P)+2\,|C_3(\alpha,P)|^2}{\left(1-|C_3(\alpha,P)|^2\right)}-\frac{C_1(P)+C_1^*(P)+2\,|C_1(P)|^2}{\left(1-|C_1(P)|^2\right)}\right)\frac{\bar{p}_{i\mu}p_{i\nu}}{2\,|\vec{p}_i|^2}$$
$$\left.\left.+\left(\frac{C_3(\alpha,P)+C_3^*(\alpha,P)+2\,|C_3(\alpha,P)|^2}{\left(1-|C_3(\alpha,P)|^2\right)}\right)\frac{(1-\alpha)}{4\,|\vec{p}_i|^2}\left(p_{i\mu}p_{i\nu}-2p_{i0}\delta_{\mu 0}p_{i\nu}\right)\right]\right\}.\qquad(4.17)$$

It is interesting to note at this point that the combinations of $C_1(P)$ and $C_3(\alpha,P)$, in the expression (4.17), are real. So even if these parameters are complex the modifications introduced by them are real. Then, after knowing that complex parameters essentially produce the same result as the real ones, they will be considered as real in what follows.

In the expression (4.17) the addition is changed by an integration, considering that $V\to\infty$, in accordance with the relation

$$\frac{1}{V}\sum_{\vec{p}_i,|\vec{p}_i|<P}=\frac{1}{(2\pi)^3}\int_0^P dp\;p^2\int_0^\pi\sin\theta d\theta\int_0^{2\pi}d\varphi,\qquad(4.18)$$

and the integrals are calculated.

After that, the parameters introduced were considered behaving in following form, for $P \sim 0$,

$$\begin{aligned} C_1(P) &\sim 1 - \frac{C_1}{2} P^2, \; C_1 > 0, \\ C_3(\alpha, P) &\sim 1 - \frac{C_3(\alpha)}{2} P^2, \; C_3(\alpha) > 0. \end{aligned} \tag{4.19}$$

The result obtained for the expression (4.17), in the limit $P \to 0$, is then

$$\exp\left\{ \int d^4x d^4y J^{\mu,a}(x) J^{\nu,a}(y) \left[\frac{g_{\mu\nu}}{(2\pi)^2 C_1} + \frac{1}{3(2\pi)^2} \left(\frac{1}{C_3(\alpha)} - \frac{1}{C_1} \right) (g_{\mu\nu} + 2\delta_{\mu 0}\delta_{\nu 0}) \right. \right. \\ \left. \left. - \frac{(1-\alpha)}{(2\pi)^2 6 C_3(\alpha)} (g_{\mu\nu} + 2\delta_{\mu 0}\delta_{\nu 0}) \right] \right\}. \tag{4.20}$$

The constant $C_3(\alpha)$ is fixed in order to cancel the terms proportional to $\delta_{\mu 0}\delta_{\nu 0}$, to make explicit the Lorentz covariance. That's why this non-physical part was introduced in the vacuum state. Then $C_3(\alpha)$ is determined by the expression

$$C_3(\alpha) = \frac{(1+\alpha)}{2} C_1,$$

and (4.20) takes the form

$$\exp\left\{ \int d^4x d^4y J^{\mu,a}(x) J^{\nu,a}(y) \frac{g_{\mu\nu}}{(2\pi)^2} \frac{1}{C_1} \right\}. \tag{4.21}$$

In the expression (4.21) is important to note that the term in the exponential is real and non-negative (4.19). And that it is independent of the gauge parameter, which is a very interesting result and it was assumed in a previous work [13].

Considering the above remarks and defining in the generating functional modification an alternative nonnegative constant $C = \frac{2(2\pi)^2}{C_1}$ (which will be called condensate parameter) for further convenience, the expression (4.21) takes the form

$$\exp\left\{ \int d^4x d^4y \sum_{a=1}^{8} J^{\mu,a}(x) J^{\nu,a}(y) \frac{g_{\mu\nu} C}{(2\pi)^4 2} \right\}.$$

For the above construction the vacuum in the zero momentum limit has the form

$$\mid \Psi \rangle = \exp\left[\sum_{a=1}^{8} \frac{1}{2} A_{0,1}^{a+} A_{0,1}^{a+} + \frac{1}{2} A_{0,2}^{a+} A_{0,2}^{a+} + B_0^{a+} A_0^{L,a+} + i \bar{c}_0^{a+} c_0^{a+} \right] \mid 0 \rangle. \tag{4.22}$$

Then in the zero momentum limit this state has exactly the same form of the one proposed previously [14, 15], the only difference appearing is introduced in the way in which the zero momentum limit is taken. That previous work [14, 15] corresponds to the particular case in which $\alpha = 1$, $C_3 (\alpha = 1) = C_1$.

The modification to the ghost generating functional of the free-particle Green functions is the same calculated previously [14, 15]. It is because the ghost sector in the vacuum state remains unchanged. As was mentioned there, for the value $C_3 (\alpha, 0) = 1$ as was fixed here, there is no modification of the ghost sector and its generating functional and propagator remain the same of the usual PQCD. The consequence of this unchanged sector was mentioned in Section 3.2, that is it guarantees the fulfillment of a Ward identity in the one loop calculation for the gluon self energy.

Then, after all the previous analysis, it is concluded that the vacuum state introduced (4.22) modifies the usual perturbation theory only through a change in the gluon generating functional of the free-particle Green functions in such a way that the total gluon propagator (including the usual perturbative piece) takes the form

$$\widetilde{D}_{\mu\nu}^{ab}(x-y) = \int \frac{d^4k}{(2\pi)^4} \delta^{ab} \left[\frac{1}{k^2} \left(g_{\mu\nu} - (1-\alpha) \frac{k_\mu k_\nu}{k^2} \right) - iC\delta\left(k\right) g_{\mu\nu} \right] \exp\left\{-ik\left(x-y\right)\right\}. \tag{4.23}$$

in which the first term is the usual perturbative propagator.

Finally it is possible to analyze the consequences of the present generalization on previous calculations, for $\langle G^2 \rangle$, quark and gluon masses, done in Chapter 3.

For the mean value of G^2 and the quarks masses, after performing calculations like the ones realized previously, was found that the results are exactly the same obtained before, independently of the α parameter values. However for the gluon mass is obtained the following result

$$m_G^2 = -\frac{3\left(1+\alpha\right) g^2 C}{(2\pi)^4}.$$

This result, α dependent, does not represent a contradiction because the gluon mass is tachyonic for all the admissible values of the α parameter (the non-negative ones), then it is not a physical magnitude and could depend on the gauge choice. However at the present state of the analysis the general proof of the gauge invariance of the theory is lacking and will be considered in future works.

Chapter 5

Summary

This work considers the extension of the former ones [13]-[15]. The results obtained can be summarized as follows:

1) The elements of the modified perturbative expansion for QCD initiated in [13] were specified. The diagram technique embodies non-perturbative information associated to a non-trivial vacuum in which exists a condensate of soft gluon pairs, as found in [14, 15].

2) The former calculation of the mean value of G^2 was completed by including all corrections of order g^2, disregarding the terms not including the condensate effects. The result was the same obtained previously in the tree approximation [13, 14]. The mean value of G^2 is a relevant quantity for the description of the QCD vacuum [17], and has been calculated previously considering non-perturbative methods. The condensate parameter g^2C was determined to reproduce the accepted value of $\langle g^2G^2\rangle$.

3) The contribution of the condensate to the gluon self-energy, up to the g^2 order, was calculated. A tachyonic result for the gluon mass was obtained, for the parameter values employed in the construction of the initial state defined in [14, 15]. This because the condensate parameter C was determined as real and non-negative in that work. The polarization of the tachyonic mode is transverse. As remarked before there are recent arguments aiming at the possibility of the existence of a tachyonic gluon mass [26, 27], and its role in improving the quark antiquark potential in bound state models of mesons [27].

4) An evaluation for the quark masses, under the influence of the condensate, was done. Various solutions were obtained for the masses of these particles, among which only one is real for all the values the current mass parameter. Employing this particular solution some interesting predictions were derived. In particular values of near one third of the nucleon mass, for the light quark masses, were obtained. Further, the effective quark masses were evaluated as determined by the known current mass for each flavor. By mean of these values, the ground state energies of some hadron resonances, reported in [11], were estimated through the simple addition of the masses of their known constituent quarks. It could be remarked

that the results obtained are reasonable good ones in comparison with the experimental data, after taking into account the high errors in the experimentally determined Lagrangian masses. This makes possible to sustain the hypothesis about the almost glue nature of the constituent mass values of the light quarks. The small contribution of the binding potential energy of such resonances is also indicated. This conclusion then supports the applicability of non-relativistic approximations in their study.

5) The effective potential was determined, in the approximation considered in a previous work [13], as a function of $\langle G^2 \rangle$. As it was obtained previously [13] the condensate is generated spontaneously from the zero field vacuum, as in the Savvidy Model. That is, it was shown that the effective action has a minimum for a mean value of G^2 different from zero.

It is worth resuming also the main aspects of the modified expansion that have been clarified after the initial work [13]:

A) The Feynman expansion, depending on the condensate parameter and a gauge parameter $\alpha = 1$, corresponds to the Wick expansion around a physical state of the free QCD.

B) Also in [14, 15] the condensation parameter C was defined as real and positive. This result determined the tachyonic value for the gluon mass evaluated here and the masses for up and down quarks as 1/3 of the nucleon mass.

C) Another aspect which is important to underline is that the selection of the parameters in the initial state [14, 15] was designed also to impose the absence of a modification for the free ghost propagator. However, as it followed from the present analysis, this property in turns is related with the fact that a condensate modification of the ghost propagator can produce a longitudinal contribution to the self-energy. But, such a term should not exist for the transversality condition of the polarization operator (Ward identity) to be obeyed. Thus, its appearance could break manifestly the gauge invariance. Therefore, the present work is also given foundation to the non-modified ghost propagator choice considered in [13]-[15].

5) Finally, it was explored the generalization of the theory for arbitrary values of the α gauge parameter. A particular vacuum state for a Perturbative QCD was proposed, it is formed by a coherent superposition of zero momentum gluon pairs, and it is a generalization of the one proposed in a previous work [14, 15]. It was analyzed that the proposed state satisfies the BRST physical conditions imposed by the operational quantization method for gauge fields, developed by Kugo and Ojima [20, 21]. The modifications in the perturbation theory introduced by the proposed vacuum state were calculated for arbitrary values of the α gauge parameter. The parameters introduced in the definition of the vacuum state were fixed in such a way that they reproduce the modification to the gluon propagator initially proposed and based in a previous work [13] starting from a functional analysis. The condensation parameter C, that modifies the standard propagator multiplying a term containing a delta function in the four-momentum p, was determined to be real and nonnegative. The consequences of the generalized theory on previously calculated magnitudes were analyzed. For the calculation of the mean value of G^2 and the quark masses was obtained that the results are identical to the ones determined for $\alpha = 1$. The expression for the gluon mass was also calculated and it is determined to be gauge

dependent and tachyonic for all the values of the α parameter. The general proof of the gauge invariance of the theory is lacking and will be study later on.

At the present stage some possible lines for the extension of the work can be visualized:

1) To explore in more detail the various possibilities for the quantization of the zero modes of the gluon field. It could allow to bypass the employment of the zero momentum limit used in the construction of the initial state, and also perhaps permit to simplify its introduction and the study of alternatives for the modified expansion describing different physical ground states.

2) Improve the study of the effective action as a functional of the condensate parameter, in order to search for a variant of the leading logarithm model useful for the investigation of field configurations associated to inter-quark strings, nucleon-nucleon potentials, etc.

3) Start the introduction of the field and coupling constant renormalization in the scheme. This would allow to investigate if the infrared behavior of the running coupling of the PQCD can be regularized by the modified expansion. The evolution of the masses at high energy will be another task to be worth considering. At last the determination of the absolute value of the constant C would be also possible after finding the running coupling evolution.

4) To investigate the possibility for a derivation of existing successful bounded state models for the heavy quark mesons [28]-[30], as realized by the a ladder approximation for the Bethe-Salpeter equation within the proposed modified expansion. The presence of a tachyonic gluon propagator in the approach (which is argued to have the effect of introducing a linearly rising component in the inter-quark potential [27]) and the obtained constituent values for the light quark masses already support such a possibility.

Appendix A

Diagrams for the evaluation of $\langle G^2 \rangle$

For the exposition of diagrams appearing in the $\langle G^2 \rangle$ calculation, only the gluon and quarks will be taken into account. The terms related with ghosts do not contribute. This conclusion can be extracted easily following the diagrams related with quarks, since the associated ghost diagrams are similar. All the contributions to $\langle G^2 \rangle$ of diagrams not including the condensate parameter are disregarded, as was justified in the introduction.

Expanding the exponential in the vertices in (2.10), up to order g^2 for the gluon and quark contributions it is obtained

$$\begin{aligned}
Z\left[J,\bar{\xi},\xi\right]=N\Bigg[1+\frac{S^G_{abc}}{3!i^2}\frac{\delta}{\delta J_a}\frac{\delta}{\delta J_b}\frac{\delta}{\delta J_c}+\frac{S^G_{abcd}}{4!i^3}\frac{\delta}{\delta J_a}\frac{\delta}{\delta J_b}\frac{\delta}{\delta J_c}\frac{\delta}{\delta J_d}+\frac{S^Q_{iaj}}{i^2}\frac{\delta}{\delta\bar{\xi}_i}\frac{\delta}{\delta J_a}\frac{\delta}{\delta\left(-\xi_j\right)}\\
+\frac{1}{2}\Bigg(\frac{S^G_{abc}}{3!i^2}\frac{\delta}{\delta J_a}\frac{\delta}{\delta J_b}\frac{\delta}{\delta J_c}\frac{S^G_{def}}{3!i^2}\frac{\delta}{\delta J_d}\frac{\delta}{\delta J_e}\frac{\delta}{\delta J_f}+\frac{S^Q_{iaj}}{i^2}\frac{\delta}{\delta\bar{\xi}_i}\frac{\delta}{\delta J_a}\frac{\delta}{\delta\left(-\xi_j\right)}\frac{S^Q_{kbl}}{i^2}\frac{\delta}{\delta\bar{\xi}_k}\frac{\delta}{\delta J_b}\frac{\delta}{\delta\left(-\xi_l\right)}\\
+2\frac{S^G_{abc}}{3!i^2}\frac{\delta}{\delta J_a}\frac{\delta}{\delta J_b}\frac{\delta}{\delta J_c}\frac{S^Q_{idj}}{i^2}\frac{\delta}{\delta\bar{\xi}_i}\frac{\delta}{\delta J_d}\frac{\delta}{\delta\left(-\xi_j\right)}\Bigg)\Bigg]\,Z_0\left[J,\bar{\xi},\xi\right], \qquad \text{(A.1)}
\end{aligned}$$

where the norm (N) up to order g^2 is given by the expression

$$\begin{aligned}
N=1-\Bigg\{\Bigg[\frac{S^G_{abcd}}{4!i^3}\frac{\delta}{\delta J_a}\frac{\delta}{\delta J_b}\frac{\delta}{\delta J_c}\frac{\delta}{\delta J_d}\\
+\frac{1}{2}\Bigg(\frac{S^G_{abc}}{3!i^2}\frac{\delta}{\delta J_a}\frac{\delta}{\delta J_b}\frac{\delta}{\delta J_c}\frac{S^G_{def}}{3!i^2}\frac{\delta}{\delta J_d}\frac{\delta}{\delta J_e}\frac{\delta}{\delta J_f}+\frac{S^Q_{iaj}}{i^2}\frac{\delta}{\delta\bar{\xi}_i}\frac{\delta}{\delta J_a}\frac{\delta}{\delta\left(-\xi_j\right)}\frac{S^Q_{kbl}}{2!i^2}\frac{\delta}{\delta\bar{\xi}_k}\frac{\delta}{\delta J_b}\frac{\delta}{\delta\left(-\xi_l\right)}\\
+2\frac{S^G_{abc}}{i^2}\frac{\delta}{\delta J_a}\frac{\delta}{\delta J_b}\frac{\delta}{\delta J_c}\frac{S^Q_{idj}}{2!i^2}\frac{\delta}{\delta\bar{\xi}_i}\frac{\delta}{\delta J_d}\frac{\delta}{\delta\left(-\xi_j\right)}\Bigg)\Bigg]\,Z_0\left[J,\bar{\xi},\xi\right]\Bigg\}_{J,\bar{\xi},\xi=0}. \qquad \text{(A.2)}
\end{aligned}$$

Now, the diagrams generated by the action of the functional differential operator in (3.3) on (A.1), after taking into account (A.2), are analysed.

The third term in the expansion (3.3) is already of order g^2. Up to this order also, it will only contribute with the first term in the expansion (A.1), which only generates the diagram shown in Fig. A.1.

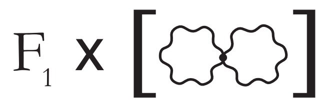


Figure A.1: Diagram 1.

Substituting the expressions for propagators and vertices in the diagram of the Fig. A.1, disregarding the terms in which the condensation parameter C do not appear and considering that the integrals of the form

$$I = \int \frac{d^D q}{(-q^2)^\alpha} = 0 \qquad \text{for} \ \ \alpha > 0 \tag{A.3}$$

within dimensional regularization, the following result for the first diagram follows:

$$-\frac{1}{8}\left[\frac{576 g^2 C^2}{(2\pi)^8}\int d^4x\right], \tag{A.4}$$

where $\int d^4x$ means the space-time volume. The symmetry factor of (A.3) is $F_1 = -\frac{1}{8}$. This diagram represents the tree approximation for the evaluation of the mean value of G^2 and it was calculated before in [13].

The second term in the expansion (3.3) contributes with the second and the fourth terms in the expansion (A.1).

With the second term of (A.1) two topologically independent diagrams appear which are represented in Fig A.2.

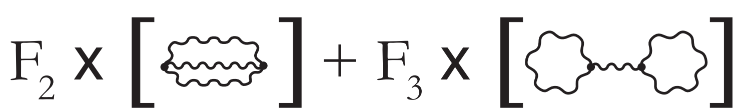


Figure A.2: Diagram 2.

With the fourth term of (A.1), one diagram appears which is represented in Fig A.3.

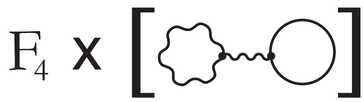


Figure A.3: Diagram 3.

For the calculation of the first diagram in Fig. A.2, when it is taken into account the $i\epsilon$ prescription for the gluon propagator, is obtained a null contribution.

The second diagram in Fig. A.2 and the one in Fig. A.3 generate a null contribution to the mean value of G^2. This is due to the appearance of terms of the type $f^{abc}\delta^{bc}$ and $T^{ij,a}\delta^{ij}$ respectively which are equal to zero due to the antisymmetry of the structure constants f^{abc} (similar arguments are used for the analogous diagrams having ghosts) and also due to the vanishing traces of the matrices $T^{ij,a}$.

Next, the first term in the expansion (3.3), up to the order g^2, contributes with the first, third, fifth, sixth and seventh of the expansion (A.1), as well as with the four terms of (A.2).

With the first term of (A.1), the diagram represented in Fig. A.4 is obtained.

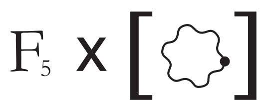


Figure A.4: Diagram 4.

With the third term of (A.1) and the first of (A.2), the diagram shown in Fig. A.5 is obtained. It can be observed that the net effect of the normalization factor of the generating functional has been to cancel out the vacuum diagrams, the same occurs for the diagrams which follow.

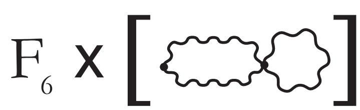


Figure A.5: Diagram 5.

With the fifth term of (A.1) and the second of (A.2), the three diagrams represented in Fig. A.6 appear; with the sixth term of (A.1) and the third of (A.2), the diagrams represented in Fig. A.7 are obtained; finally for the action of (3.3) on the seventh term of (A.1) and the fourth of (A.2) it follows Fig. A.8.

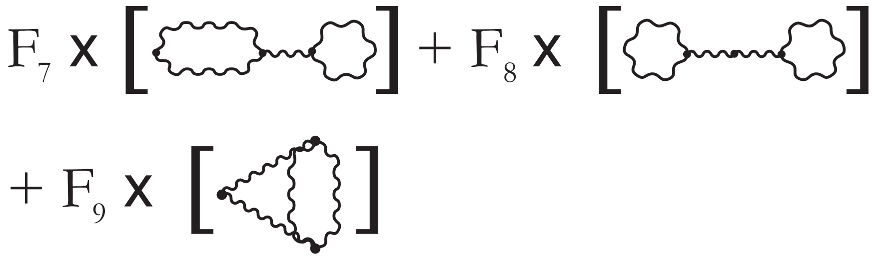


Figure A.6: Diagram 6.

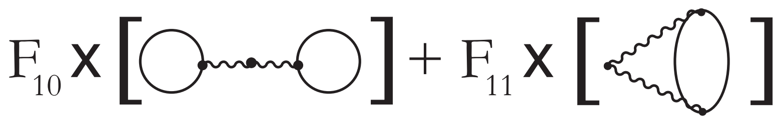


Figure A.7: Diagram for 7.

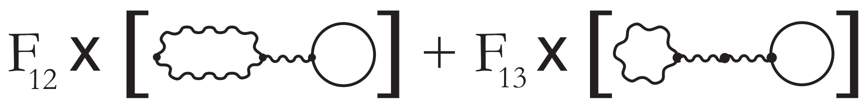


Figure A.8: Diagram 8.

Once, all the diagrams generated by the first term in the expansion (3.3) were examined, the results for their contributions are analyzed.

For the diagram represented in Fig. A.4, in the momentum representation it is obtained:

$$\int \frac{d^4k}{(2\pi)^8} \delta^{a_i a_j} \left[- \left(g_{\mu_i \mu_j} k^2 - k_{\mu_i} k_{\mu_j} \right) \right] \delta^{a_i a_j} g^{\mu_i \mu_j} \left(-iC\delta\left(k\right) \right) \int d^4x = 0, \tag{A.5}$$

Thanks to the antisymmetry property of the structure constants and the vanishing traces of the matrices $T^{ij,a}$, the first and second diagrams of Fig. A.6, the first of Fig. A.7 and the two of the Fig. A.8 will also vanish. In this way, it is only needed to calculate the contributions of the following diagrams: the one in Fig. A.5, the third of Fig. A.6 and the second of Fig. A.7. After considering dimensional regularization and the $i\epsilon$ prescription it was found a vanishing result for the contribution of that diagrams.

It is important to remark that the same occurs for the diagrams generated by the ghost particles. Then the only diagram contributing to the mean value of G^2, in the considered approximation, is the one in Fig. A.1.

Appendix B

Longitudinal and Scalar Modes contribution

The longitudinal and scalar modes contribution is determined by the expression

$$
\begin{aligned}
&\langle 0 \mid \exp\left[C_3^*\left(\alpha,P\right)B_{\vec{p}_i}^{a}A_{\vec{p}_i}^{L,a}\right]\exp\left\{i\int d^4xJ^{\mu,a}\left(x\right)\left(A_{\vec{p}_i}^{L,a+}f_{p_i,L,\mu}^*\left(x\right)+\right.\right.\\
&\qquad\left.\left.+B_{\vec{p}_i}^{a+}\left[f_{p_i,S,\mu}^*\left(x\right)+\left(1-\alpha\right)\left(A^*\left(p_0,x_0\right)f_{p_i,L,\mu}^*\left(x\right)+B_{\mu}^*\left(p_i,x\right)\right)\right]\right)\right\}\\
&\times\exp\left\{i\int d^4xJ^{\mu,a}\left(x\right)\left(B_{\vec{p}_i}^{a}\left[f_{p_i,S,\mu}\left(x\right)+\left(1-\alpha\right)\left(A\left(p_0,x_0\right)f_{p_i,L,\mu}\left(x\right)+B_{\mu}\left(p_i,x\right)\right)\right]+\right.\right.\\
&\qquad\left.\left.+A_{\vec{p}_i}^{L,a}f_{p_i,L,\mu}\left(x\right)\right)\right\}\exp\left[C_3\left(\alpha,P\right)B_{\vec{p}_i}^{a+}A_{\vec{p}_i}^{L,a+}\right]\mid 0\rangle. \qquad \text{(B.1)}
\end{aligned}
$$

Using the same methodology of the previous work [14, 15], is first calculated

$$
\begin{aligned}
&\exp\left\{i\int d^4xJ^{\mu,a}\left(x\right)\left(B_{\vec{p}_i}^{a}\left[f_{p_i,S,\mu}\left(x\right)+\left(1-\alpha\right)\left(A\left(p_0,x_0\right)f_{p_i,L,\mu}\left(x\right)+B_{\mu}\left(p_i,x\right)\right)\right]+\right.\right.\\
&\qquad\left.\left.+A_{\vec{p}_i}^{L,a}f_{p_i,L,\mu}\left(x\right)\right)\right\}\exp\left[C_3\left(\alpha,P\right)B_{\vec{p}_i}^{a+}A_{\vec{p}_i}^{L,a+}\right]\mid 0\rangle\\
&=\exp\left\{C_3\left(\alpha,P\right)\left(B_{\vec{p}_i}^{a+}-i\int d^4xJ^{\mu,a}\left(x\right)f_{p_i,L,\mu}\left(x\right)\right)\times\right. \qquad \text{(B.2)}\\
&\quad\times\left.\left(A_{\vec{p}_i}^{L,a+}-i\int d^4xJ^{\mu,a}\left(x\right)\left[f_{p_i,S,\mu}\left(x\right)+\left(1-\alpha\right)\left(A\left(p_0,x_0\right)f_{p_i,L,\mu}\left(x\right)+B_{\mu}\left(p_i,x\right)\right)\right]\right)\right\}\mid 0\rangle,
\end{aligned}
$$

in the same way can be calculated the left part of (B.1).

Substituting the right (B.2) and left parts in (B.1), and introducing the following notation,

$$\begin{aligned}
C^* &\equiv C_3^*(\alpha,P), \quad C\equiv C_3(\alpha,P),\\
\hat{A}^+ &\equiv A^{L,a+}_{\vec{p}_i},\ \hat{A}\equiv A^{L,a}_{\vec{p}_i}, \quad \hat{B}^+\equiv B^{a+}_{\vec{p}_i},\ \hat{B}\equiv B^{a}_{\vec{p}_i},\\
a_1 &\equiv -i\int d^4xJ^{\mu,a}(x)\left[f^*_{p_i,S,\mu}(x)+(1-\alpha)\left(A^*(p_0,x_0)\,f^*_{p_i,L,\mu}(x)+B^*_\mu(p_i,x)\right)\right],\\
a_2 &\equiv -i\int d^4xJ^{\mu,a}(x)\left[f_{p_i,S,\mu}(x)+(1-\alpha)\left(A(p_0,x_0)\,f_{p_i,L,\mu}(x)+B_\mu(p_i,x)\right)\right],\\
b_1 &\equiv -i\int d^4xJ^{\mu,a}(x)\,f^*_{p_i,L,\mu}(x),\\
b_2 &\equiv -i\int d^4xJ^{\mu,a}(x)\,f_{p_i,L,\mu}(x),
\end{aligned}\tag{B.3}$$

the expression (B.1) takes the form

$$\langle 0\mid\exp\left\{C^*\left(\hat{A}+a_1\right)\left(\hat{B}+b_1\right)\right\}\exp\left\{C\left(\hat{B}^++b_2\right)\left(\hat{A}^++a_2\right)\right\}\mid 0\rangle.\tag{B.4}$$

Following exactly the same procedure described previously [14, 15], the following recurrence relation is obtained for (B.4).

$$\begin{aligned}
&\exp\left\{C^*a_1b_1+C\left(C^*a_1-a_2\right)\left(C^*b_1-b_2\right)\sum_{m=0}^{n}\left[|C|^{2(2m)}+|C|^{2(2m+1)}\right]\right\}\\
&\langle 0\mid\exp\left\{C^*\hat{A}\hat{B}+C^{*n+1}C^{n+1}\left(\left(C^*b_1-b_2\right)\hat{A}+\left(C^*a_1-a_2\right)\hat{B}\right)\right\}\exp\left\{C\hat{B}^+\hat{A}^+\right\}\mid 0\rangle.
\end{aligned}\tag{B.5}$$

To take the limit $n\to\infty$ it is considered $|C|<1$, which implies the expression (B.5) takes the form

$$\begin{aligned}
&\exp\left\{C^*a_1b_1+C\left(C^*a_1-a_2\right)\left(C^*b_1-b_2\right)\frac{1}{\left(1-|C|^2\right)}\right\}\\
&\langle 0\mid\exp\left\{C^*\hat{A}\hat{B}\right\}\exp\left\{C\hat{B}^+\hat{A}^+\right\}\mid 0\rangle.
\end{aligned}\tag{B.6}$$

Finally the notation (B.3) is substituted in (B.6). After that all the functions of $\vec{p}_i$ are expanded in the vicinity of $\vec{p}_i=0$, keeping in mind that the sources are located in a space finite region, it is necessary to consider only the first terms in all the expansion. Then, for the expression (B.6), the result obtained is (4.16).

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